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Editorial

Instability and fluctuations in intertemporal equilibrium models: Presentation

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Abstract

This paper introduces the special issue of the *Journal of Mathematical Economics on Instability and Fluctuations in Intertemporal Equilibrium Models*.

Keywords: Instability; Fluctuations; Intertemporal equilibrium models

Over the last 30 years, intertemporal equilibrium models have become a central theoretical framework in the analysis of macroeconomic fluctuations. Since the basic formulations of the infinite-horizon growth model by Ramsey (1928) and the overlapping generations model with capital accumulation by Diamond (1965), many extensions and generalizations of these standard environments have been provided by the literature. This special issue of the *Journal of Mathematical Economics* contains papers at the cutting edge of research on *Instability and Fluctuations in Intertemporal Equilibrium Models*, which present some of the most recent developments in macroeconomics.

While many of the papers in this special issue retain the standard (discounted utilitarian) representation of intertemporal preferences, a first set of contributions explore the implications of alternative intertemporal preference structures, which still fall within the framework of the seminal work of Koopmans (1960). A special emphasis is put on impatience and the rate of time preference.

In the paper "*On the impatience implications of Paretian social welfare functions*", Banerjee and Mitra (2007) investigate the impatience implications resulting from the assumption of existence of a Paretian social welfare function aggregating infinite utility streams. They show that such aggregator must exhibit a preference for advancement of the timing of utility from the future to the present, generally known as *impatience*. The impatience phenomenon can be shown to be widespread in a very general framework: the set of utility streams which exhibit impatience has the same power as the entire set of utility streams. In a more specialized (but standard) framework, the set of utility streams at which the social welfare function exhibits impatience, is *dense* in the set of all utility streams.

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Drugeon and Wigniolle (2007), in their paper "*On time preference, rational addiction and utility satiation*", study the implications for intertemporal consumer choice, when the consumer's utility functional is an integral of intertemporal utilities, in which present consumption has a positive effect, and past consumption has a negative effect on the intertemporal utilities, the latter effect being captured (as in Becker and Murphy, 1988) through an aggregate variable representing *consumption habits*. It is shown that two types of steady states may arise: one with utility satiation, and the other an interior unsatiated state. The phenomenon of *addiction*, which occurs when past consumption positively influences current consumption, is seen to arise near the unsatiated steady state when the rate of time preference is an increasing function of consumption habits. When addiction is sufficiently strong, the unsatiated steady state becomes unstable, and the only admissible stationary position is the satiated steady state.

A second set of contributions is concerned with the analysis of existence, uniqueness and stability of equilibrium in aggregate infinite horizon models with heterogeneous households. The heterogeneity concerns preferences, discount factors and/or capital and labor endowments.

In the paper "*Equilibrium dynamics in an aggregative model of capital accumulation with heterogeneous agents and elastic labor*", Le Van et al. (2007) examine whether and under which conditions the usual dynamic properties hold when the one-sector infinite-horizon Ramsey (1928) model is extended to include heterogeneous households and an endogenous primary factor such as labor. The monotonicity property does not carry over if one permits many consumers with different discount factors. The convergence of the optimal capital sequence to a particular stock is still true, but that stock is not itself a steady state. Moreover, all consumers do not necessarily supply labor at any period.

Following the conjecture initially formulated by Ramsey (1928), Becker (1980) demonstrated that, in a one-sector discrete time model with agents forbidden to borrow against their future labor income, in the long-run stationary state the economy's capital is concentrated in the most patient household. The assumed competitiveness of the capital market however appears to be problematic since the most patient individual has all the capital, yet behaves competitively, taking the rental price of capital as given and uninfluenced by its accumulation (or decumulation) decisions. In "*Strategic Ramsey equilibrium dynamics*", Becker and Foias (2007) explores the dynamic properties of a strategic model with many agents in the case where only the most patient one holds capital in the steady state. He shows that there is a class of economies for which the Turnpike Property holds.

A third set of contributions still considers infinite horizon models with heterogenous agents, i.e., households or countries, but focusses on the existence of multiple equilibria, i.e., local indeterminacy, and the occurrence of expectations-driven fluctuations. In presence of market imperfections, local uniqueness of the equilibrium is indeed no longer ensured.

Lloyd-Braga and Modesto (2007), in their paper "*Indeterminacy in a finance constrained unionized economy*", investigate how unions affect the emergence of stochastic and endogenous fluctuations. They depart from a model with heterogenous agents and financial constraints, introduced by Woodford (1986), and consider three additional distortions: there are social increasing returns in production caused by capital and labor externalities, wages and employment are determined each period through a bargaining process between unions and firms, and there is a positive real reservation wage that leads to underemployment equilibria. They find that the influence of unions on indeterminacy depends crucially on the joint configuration of union power and technology parameters. In particular, for small externalities and some elasticity of capital-labor substitution above unity, union power facilitates the occurrence of indeterminacy.

In a recent contribution, Ghiglino and Olszak-Duquenne (2005) have studied the effect of wealth inequality on the occurrence of local indeterminacy in a general equilibrium model with heterogeneous agents and technological externalities. Building on a similar formulation but considering several countries, Ghiglino (2007), in "*Trade, redistribution and indeterminacy*", focusses on a tractable two-sector growth model in which the technology is specified as in Boldrin and Rustichini (1994). He shows that indeterminacy may occur due to the enlargement of the markets for goods and factors. In particular, the integration into a common market on which countries trade the produced good and the inputs may lead to indeterminacy even when the equilibrium under full autarchy is determinate.

By introducing externalities into a multi-sector endogenous growth model, Benhabib et al. (2000) provided a factor intensity condition under which indeterminacy may occur. Doi et al. (2007), in "*A two-country dynamic model of international trade and endogenous growth: multiple balanced growth paths and stability*", extend this formulation to a two-country endogenous growth model which explains joint determination of long-run trade patterns and world growth rates. They prove the existence and local stability of the continuum of balanced growth paths, and show that the main standard trade propositions hold under some modifications.

Local indeterminacy of equilibria derived from the presence of externalities is also analyzed in a fourth set of contributions but under the assumption of a representative infinitely lived agent. The focus is now oriented towards the existence of endogenous growth or the consideration of multi-sector productive systems.

Bosi and Nourry (2007), in "*Animal spirits and public production in slow growth economies*", consider a discrete time version of the endogenous growth model developed by Barro (1990), but augmented in order to consider public production. Public dividends are used to finance the public good and the possibility of a partial depreciation of the public good is taken into account. They show that for low levels of economic growth, large substitution effects rule out the occurrence of endogenous fluctuations. Conversely, macroeconomic volatility arises under dominant income effects, as soon as the share of the public sector in total production is low enough.

In one-sector models local indeterminacy is known to require some increasing returns based on externalities, a strongly elastic labor supply, and a large enough elasticity of intertemporal substitution in consumption. On the contrary, as proved in Benhabib and Nishimura (1998), in two-sector models with linear utility function and sector-specific externalities, local indeterminacy is compatible with constant returns at the social level and does not require any restriction on the labor supply. In the paper "*Indeterminacy in discrete-time infinite-horizon models with non-linear utility and endogenous labor*", Nishimura and Venditti (2007) consider a discrete-time two-sector model with sector specific externalities, CES technologies and a CES additively separable utility function defined over consumption and leisure. They prove that, contrary to one-sector models, when labor is infinitely elastic the steady state is always saddle-point stable, and when the elasticity of intertemporal substitution in consumption is sufficiently large, local indeterminacy requires a low enough elasticity of labor.

A final set of contributions considers stochastic versions of the aggregate infinite-horizon and overlapping generations models. The production of the final good is now assumed to be affected by a stochastic technological shock. Existence, uniqueness and stability of the equilibrium are then discussed.

In the paper "*Stochastic optimal growth with bounded or unbounded utility and with bounded or unbounded shocks*", Kamihigashi (2007) studies a one-sector stochastic optimal growth model with i.i.d. productivity shocks in which utility is allowed to be bounded or unbounded, the shocks are allowed to be bounded or unbounded, and the production function is not required to satisfy the

Inada conditions at zero and infinity. The existence and stability results are obtained by extending Nishimura and Stachurski's (2005) recent arguments.

Morand and Reffett (2007), in "*Stationary markovian equilibrium in overlapping generation models with stochastic non-classical production*", develop a new monotone iterative approach for studying the questions of existence, stability, and computation of Markovian equilibrium for a large class of OLG models with non-classical stochastic production and Markov shocks. They prove the existence of Stationary Markovian Equilibrium and provide successive approximation methods for obtaining extremal "pure" Stationary Markovian Equilibrium corresponding to each Markovian equilibrium decision policy, thus directly addressing the question of stability for the class of economies under consideration through the use of constructive methods.

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